

# Similarity Networks in Smart Structure Applications

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## Abstract

Smart structures are a challenge for control system engineers due to the high number of distributed sensors and actuators needed. The control algorithms for these distributed and hierarchical control problems are currently not very well understood and new control methods have still to be developed. Neural network controllers possess the potential to learn from past examples and are thus a promising approach to the intended future intelligent control of smart structures. Using an indirect neural controller concept approach, a plant model is needed. In this paper we investigate the use of a dimensionally homogeneous neural network for nonlinear system identification and show the performance comparison to a typical neural identification technique.

**Keywords:** Neural Control, Smart Structures, Dimensional Analysis

## 1 Motivation

Originating in aerospace engineering, composite materials have found use in many industrial applications. New concepts of active materials using piezoelectric, magnetic, or thermal activation extend the idea of composite materials with incorporated actuator layers and fibers. In the sporting industry composites including piezoelectric elements, layers or fibers have already been used in control of skis, baseball bats, and bicycles [1].

Smart Structures are a challenge for control theory not only due to the distributed and hierarchical control needed, but also due to the highly nonlinear behaviour of the structure. In recent years, neural networks have been applied to nonlinear control problems [12, 13, 3, 4]. Most neural network controllers are designed as indirect controllers using a model of the plant. Even direct controllers usually can only be found using *a priori* knowledge of the plant. Dimensional analysis is a powerful tool in mechanics, where the dimensionality of a problem is reduced and so as a consequence, significant experimental effort may be saved. Combined with neural networks, the technique of dimensional analysis reduces the search space of the unknown weights, which are then restricted to syntactically correct states only. This property is shown to lead to better convergence, approximation, and generalization properties of the neural network.

## 2 Smart Structures

Smart Structures are structural elements which include active materials, such as electrostrictives, magnetostrictives, or shape memory alloys (SMAs). Other new active materials are electrorheological fluids

and hydrogels [11]. All these materials have in common, that their material properties are strongly nonlinear, and thus modelling of the structural behaviour has also to be nonlinear.

Optical astronomy and the sporting goods industry have been the first to implement smart structures in commercial applications. K2 together with ACX applies piezoelectric smart structures to skis, baseball bats, and bicycles. The European Southern Observatory ESO uses smart structures for active and adaptive optics in the new Very Large Telescope (VLT) facility in La Silla, Chile [2]. Other observatories, such as the Keck telescope on Mauna Kea, Hawaii, are also using adaptive optics. Other applications of smart structures, such as noise reduction in appliances and vehicles will follow soon.

In the context of smart structures, a high number of distributed actuators and sensors is needed to control the structural behaviour. In noise reduction the additional difficulty arises from the need of high sensing and actuation bandwidth of the sensors, actuators, and the control hard- and software. Strategies like hierarchical and distributed control are currently under investigation in control theory. A neural network control approach seems very promising, especially in the context of distributed control due to the implicit possibility of algorithm parallelization and already existing neuro-hardware.

### 3 Structural Control

A low bandwidth application of structural control is shown in figure 1, where adaptive optics are used to correct deviations in the geometric form of the primary mirror and the tube of a telescope by changing the shape of the primary mirror about once a second [2].

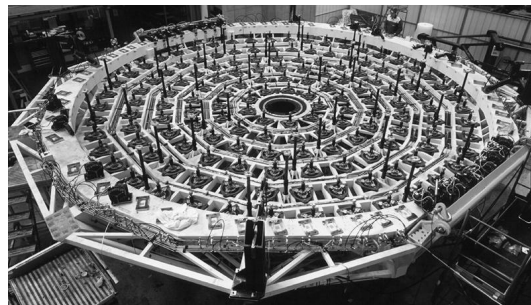


Figure 1: The Very Large Telescope main mirror active supports [2]

High bandwidth applications are active optics, where optical disturbances due to atmospheric influence are corrected [2], or vibration control of space structures [9, 14] and tall buildings [21].

Current research focuses on nonlinear control and distributed and hierarchical control. Especially with the highly nonlinear behaviour of smart structures, which are often built as fiber composites and layered structures, the linear modelling of the structure is not sufficient for control. Distributed and hierarchical control as shown in figure 2 takes advantage of the already distributed sensors and actuators used in smart structures. The distribution of control law or rules to small and fast independent units is expected to allow for high bandwidth control and for a higher fault tolerance in the overall system. A global controller synchronizes the local controllers in such a way, that low bandwidth but large area control is done by the global controller and high bandwidth and small area control is achieved by the local controller. For such a distributed control system hierarchical control laws are mostly unknown and have still to be developed.

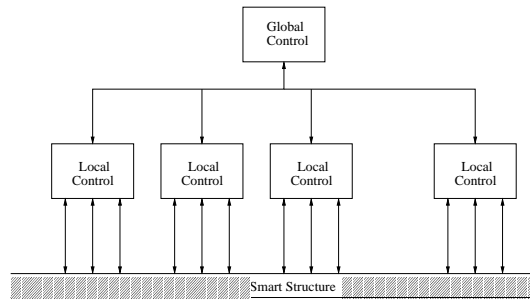


Figure 2: Hierarchical control for smart structures with embedded sensors and actuators

## 4 Principles of neural control

Neural networks, being a structured network of computational units called "neurons", are inherently well suited as a system for distributed computations. Hierarchical structures are easily imposed on neural networks and the neurons being nonlinear computational units make the neural networks suitable for nonlinear control tasks, such as in smart structure applications.

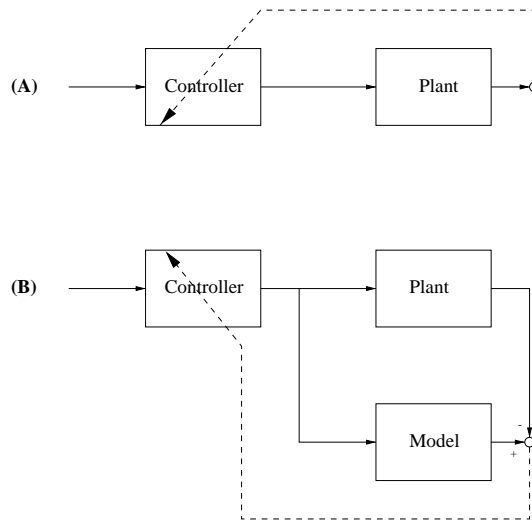


Figure 3: neural control architectures: (A) direct neural control, (B) indirect neural control

Two different methods of implementing neural network controllers are shown in figure 3. The direct method implements a neural network controller with measurements from the plant only. This black-box approach is not investigated further in this work. Using a model of the plant, a neural network controller can be learned using the indirect control method. The model used for indirect neural control can be found using standard system identification techniques, artificial intelligence techniques [20], or engineering modelling using first principles.

Combining engineering modelling techniques with artificial intelligence opens new possibilities for the generation of the plant model needed for neural control, but also for general modelling purpose. In this paper, the combination of dimensional analysis as an engineering model building technique in conjunction with the artificial intelligence technique of neural networks is shown in greater detail.

## 5 Dimensional Analysis

Based on the principle of dimensional homogeneity, dimensional analysis as a tool origins in the works of Buckingham [8] and Bridgman [7]. Based on the  $\Pi$ -Theorem of Buckingham [8], dimensionless groups (also called pi-variables) can be found from the knowledge of a relevance list only and the number of degrees of freedom in the problem can be reduced by the rank of the dimensional matrix. These dimensionless groups account each for a different physical phenomenon, and so, using intense domain knowledge, the complexity of the problem formulation can be reduced.

Traditional and new applications of dimensional analysis include fluid mechanics [10], materials science [22], chemical processes [23], and artificial intelligence [19].

## 6 System Identification using Neural Networks

Applying Buckingham's Pi-theorem to neural networks [17, 19], the possible network states are restricted to dimensionally homogeneous formulations of the input-output relationship. This reduced search space for the unknown weights results in better convergence and better approximation and generalization properties of the neural network.

In the following example we will show the identification of a fluid flow reservoir system using a conventional and a dimensionally homogeneous neural network.

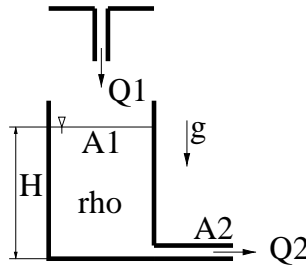


Figure 4: The reservoir system

The reservoir system shown in figure 4 can be described using the input and output mass flow  $Q_1$  and  $Q_2$ , the surface area of the reservoir  $A_1$ , the cross section of the drain  $A_2$ , the fluid mass density  $\rho$ , and the fluid level  $H$ . Deriving the governing equations for the reservoir we get the nonlinear equations [5]

$$\dot{H} = k_1\sqrt{H} + k_2Q_1 \quad (1)$$

$$Q_2 = k_3\sqrt{H} \quad (2)$$

with  $k_1 = -\frac{A_2}{A_1}\sqrt{2g}$ ,  $k_2 = \frac{1}{\rho A_1}$ , and  $k_3 = \rho\sqrt{2g}A_2$ . The output mass flow  $Q_2$  shall not be of interest here, we are asking for the equilibrium height  $H^*$  in terms of different input mass flow rates  $Q_1$ . The equilibrium height  $H^*$  is then given by the equation

$$H^* = \left(\frac{k_2}{k_1}\right)^2 Q_1^2 \quad (3)$$

Applying dimensional analysis to this problem, we get the dimensionless groups

$$\pi_1 = \frac{H}{\sqrt{A_1}} \quad \pi_2 = \frac{Q_1}{\rho\sqrt{g}A_1^{5/4}} \quad \pi_3 = \frac{A_2}{A_1} \quad (4)$$

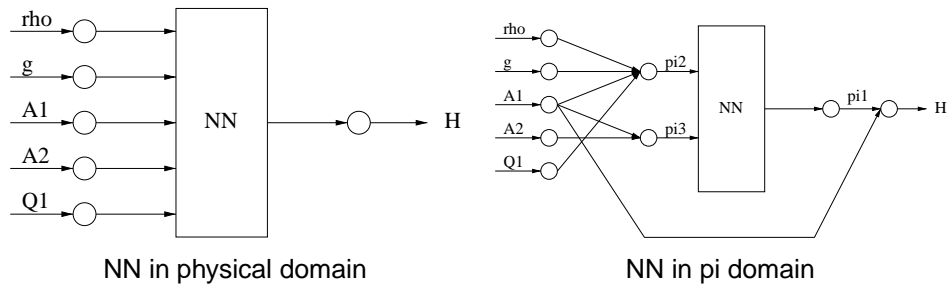


Figure 5: Architecture and training records of neural networks in the physical and the pi domain

The neural network architectures used for system identification in the physical domain as well as in the pi domain are shown in figure 5. Both networks have been trained using a gradient descent algorithm with momentum and adaptive learning rate. The network used in the physical domain has five input, ten hidden, and one output unit, the network used in the pi-domain consists of two input, four hidden, and one output neuron. A nonlinear sigmoidal activation function was used in all neurons, training was restricted to 500 epochs. The training data set consisted of 91 patterns in the range  $0 \leq Q_1 \leq 90$ , and the same data set was used for training in the physical and in the pi-domain.

In figure 6 the results of the identification using a neural network in the physical and the pi domain are shown together with the analytical solution. Please note, that the recall of the network trained in the physical domain returns even values of  $H^* < 0$ , which is physically impossible.

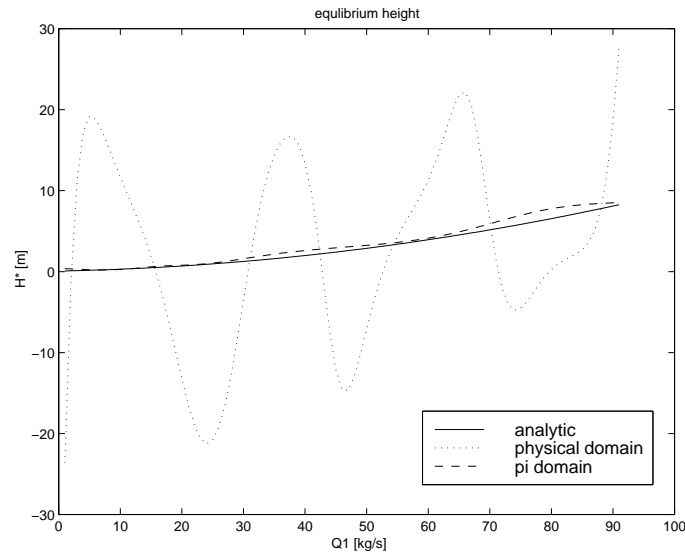


Figure 6: System identification in physical and pi domain

## 7 Summary

In this paper we have shown, that the application of neural networks in smart structure applications has very promising features. Based on Buckingham's Pi-Theorem, we have shown by means of a numerical simulation of the fluid flow reservoir system, that dimensionally homogeneous neural networks are better suited for system identification compared to neural networks trained in the physical domain. This property is very valuable for indirect neural control, and has been observed in many other numerical examples to outperform conventional neural networks.

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## A Π-Theorem

In physics often functions of the type  $x_n = f(x_1, \dots, x_{n-1})$  serve as quantitative models. An explicit function  $x_n = f(x_1, \dots, x_{n-1})$  can hereby always be written in the implicit form  $f(x_1, \dots, x_n) = 0$  (to ease the mathematical notations, the symbol  $f$  is used for both the explicit and the implicit form). Since the formal correctness of any sequence of algebraic operations on an equation can be falsified using the so-called *dimensions check*, it is clear that all possibly correct functions  $f$  in physics have to belong to the so-called class of *dimensionally homogeneous functions* [8, 10, 15, 16, 18].

Due to the property of dimensional homogeneity of all possibly correct functions in their general implicit form  $f(x_1, \dots, x_n) = 0$ , the Pi-Theorem of Buckingham [8, 10] holds in all physics and is stated in the following:

**Theorem 1 (Pi-Theorem)** *From the existence of a complete and dimensionally homogeneous function  $f$  of  $n$  physical quantities  $x_i \in \mathbb{R}^+$  follows the existence of a dimensionless function  $F$  of only  $m \leq n$  dimensionless quantities  $\pi_j \in \mathbb{R}^+$*

$$f(x_1, \dots, x_n) = 0 \quad (5)$$

$$F(\pi_1, \dots, \pi_m) = 0 \quad (6)$$

where  $m = n - r$  is reduced by the rank  $r$  of the dimensional matrix formed by the  $n$  dimensional quantities. The dimensionless quantities (also dimensionless products or dimensionless groups) have the form

$$\pi_j = x_j \prod_{i=1}^r x_i^{-\alpha_{ji}} \quad (7)$$

for  $j = 1, \dots, m \in \mathbb{N}^+$  and with the  $\alpha_{ji} \in \mathbb{R}$  as constants.

The restriction to positive values of the dimensional parameters  $x_i \in \mathbb{R}^+$  can be satisfied by coordinate transforms and is common in physics. Additionally it can be shown that modern proofs of the Pi-Theorem impose no restriction on the specific kind of the operator  $f$  and are thus valid for physical equations without exceptions [10, 6].

The so-called *dimensional matrix* associated with the *relevance list of variables*  $x_1, \dots, x_n$  is shown in the left hand side of figure 7. This dimensional matrix has  $n$  rows for the variables  $x_i$  and up to  $k$  columns

for the representation of the dimensional exponents  $e_{ij}$  of the variables  $x_i$  in the  $k$  base dimensions  $s_k$  of the employed unit system. In the current known SI-unit system seven dimensions (mass, length, time, temperature, current, amount of substance and intensity of light) are distinguished, thus  $k \leq 7$ .

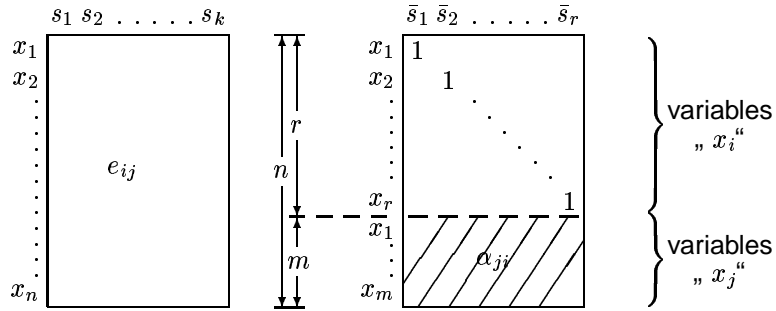


Figure 7: Definition of dimensional matrix

To calculate the dimensionless products  $\pi_j$  in equation (7), the dimensional matrix of the relevance list of variables  $x_1, \dots, x_n$  as shown in the left hand side of figure 7 needs to be created. By rank preserving operations the upper diagonal form of the dimensional matrix as shown in the right hand side of figure 7 is obtained. This means that either multiples of matrix columns may be added to each other or that matrix rows can be interchanged. The unknown exponents  $-\alpha_{ji}$  of the dimensionless products in equation (7) are then automatically determined by negation of the values of the resulting matrix elements  $\alpha_{ji}$  in the hatched part of the matrix on the lower right hand side of figure 7.