

# Identification of Tether Dynamics by means of Neural Networks during a Deployment Procedure

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## Abstract

Future utilization of the International Space Station (ISS) exhibits a demand for frequent payload return by means of small unmanned re-entry capsules. Conventional propulsive deorbit systems could be replaced by tether systems that yield high system mass savings. In order to guarantee sufficient landing accuracies, the tether deployment has to be controlled. Beside conventional methods the use of an adaptive neural controller is proposed. The present paper demonstrates the successful identification of the highly non-linear and time-variant tether dynamics, using feed-forward-networks. An operating point is selected along a predefined optimal tether deployment path, where disturbances are imposed. The accumulated deviations from the reference path are calculated by forward integration of the equations of motion. The training patterns obtained are transformed into dimensionless state space by applying the Pi-Theorem of Buckingham. The results obtained provide the basis for a future development of an indirect neural controller.

*Keywords:* tether system, re-enty capsule, frequent payload return, neural controller, system identification, similarity network, Pi-Theorem

## Introduction

The use of tethers within the gravity field of the earth is an excellent way to utilize the principles of orbit mechanics for a whole variety of applications. Among these, the tether-assisted deorbit of small re-entry capsules recently gained much interest. Simple ballistic or semi-ballistic re-entry capsules could be used to return materials processed in space combining quick access to the samples by the users on ground with low cost. The operation of winged re-entry vehicles, such as the Shuttle Orbiter, is complex and the flight rate is low. In order to enter into an elliptic transfer orbit, a specific deorbit impulse has to be applied in the opposite of the flight direction. Today this is mainly performed by means of deorbit modules equipped with propulsion systems. This conventional approach could be replaced by a tether-assisted deorbit maneuver that utilizes momentum exchange and yields high system mass savings [1]. For that purpose a long and thin tether is deployed from the space station in direction to the earth with the capsule attached. Due to orbit mechanical principles the velocity of the capsule after the tether is cut is reduced such, that it enters into a return trajectory. Unfortunately this maneuver is highly sensitive regarding disturbances, therefore the use of a controller during deployment is mandatory. The dynamics of the system are highly non-linear and time-variant. Therefore an indirect adaptive neural controller is proposed. A successful future development of such controllers strongly depends on the quality of the identification of the tether system. Therefore this paper focuses on the system identification of the tether dynamics during a deployment procedure.

## Tether-assisted Deorbit of Re-entry Capsules

There are two possible options to perform a tether-assisted deorbit maneuver, the static (hanging) and the dynamic (swing) release. By using a static release the capsule is deployed close to the vertical to a designated altitude below the station, where at an appropriate time the capsule is disconnected from the tether [2]. Due to the momentum exchange, the capsule velocity is too small compared to the velocity necessary for this orbit, and the

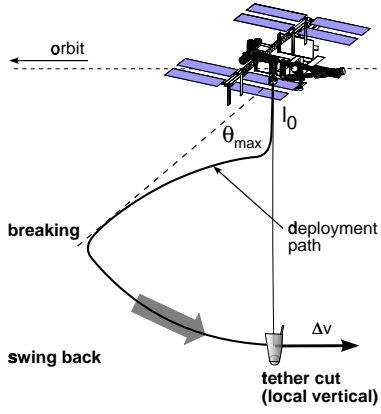


Figure 1: A dynamic release maneuver starting at a tether length  $l_0$

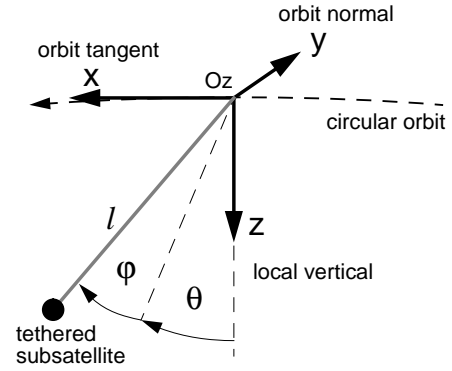


Figure 2: Coordinate system

body enters into a re-entry trajectory. The perigee reduction ( $r_0 - r_p$ ) amounts to [3]

$$r_0 - r_p \approx 7l. \quad (1)$$

Due to orbit mechanical reasons, a deployment towards the earth implies an in-plane movement in flight direction. In the case of a dynamic release the capsule is deployed along a predefined deployment path towards the maximum tether length and a particular large angle according to Fig. 1. At this point a swing back to the vertical is initiated in the opposite of the flight direction. The tether length is kept constant and the capsule is released by cutting the tether close to the local vertical. By applying a dynamic release, the tether length can be highly reduced, since the additional impulse caused by the swing back increases the perigee reduction to [1]

$$r_0 - r_p \approx 7l + 4\sqrt{3} \sin \theta_{max} \cdot l. \quad (2)$$

The dynamic release as well as the static release are very sensitive with respect to disturbances during the first deployment phase. An uncontrolled deployment causes strong deviations from the predefined landing site [1].

The examined system is composed of two point endmasses,  $m_1$  and  $m_2$ , that are connected to each other with a long, thin tether. The mass of the tether is much less than the endmasses and is thus neglected. Furthermore the tether is assumed to be straight and inextensible. Fig. 3 shows the existing forces on orbital tether systems. Each mass is affected by the gravitational force  $F_g$  as well as by the centrifugal force  $F_z$ . The resulting force pulls the mass  $m_2$  outward in radial direction and the mass  $m_1$  towards the earth. The resulting force can be split up into a radial component, which stretches the tether, and a restoring perpendicular component, that stabilizes the system in the local vertical. One should differ between the center of orbit,  $Oz$ , the center of gravity,  $Gz$  and the center of mass,  $Mz$ , on principle. They can be assumed to be situated at the orbital radius  $r_0$  [1]. Furthermore, since the mass of the space station  $m_2 \approx 415t$  is much higher than the mass of the re-entry vehicle  $m_1 = 170kg$  considered, the center of orbit is assumed to be situated at the position of the space station  $r_0 \approx r_2 = 6771km$ .

### Equations of Motion

In order to simulate a deployment procedure, the equations of motion of the system have to be derived. It is useful to describe the relative motion of the capsule with respect to the space station (center of orbit) that is assumed to move in a circular orbit. The Lagrangian of the system is given by

$$L = E_{kin} - E_{pot}, \quad (3)$$

where the kinetic energy  $E_{kin}$  and the gravitational potential energy  $E_{pot}$  (to first order) are given by [5]

$$E_{kin} = \frac{m^*}{2} (j^2 + l^2 ((\dot{\theta} - \Omega)^2 \cdot \cos^2 \varphi + \dot{\varphi}^2)), \quad (4)$$

$$E_{pot} = \frac{m^*}{2} \Omega^2 l^2 (1 - 3 \cos^2 \theta \cos^2 \varphi). \quad (5)$$

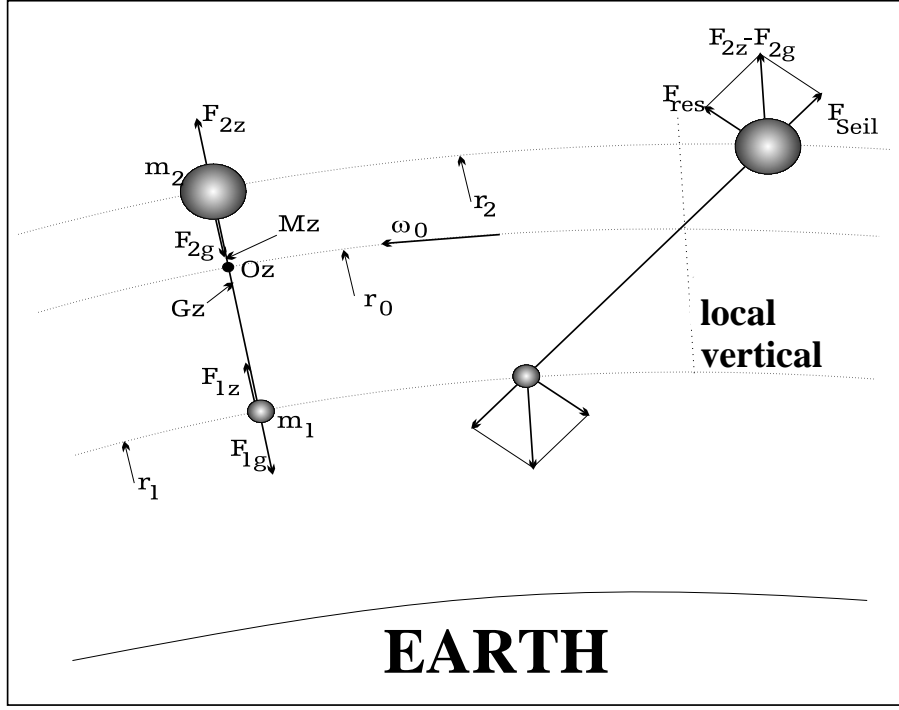


Figure 3: Forces on orbital tether systems [4]

With the assumption  $m_2 \gg m_1$  follows

$$m^* = \frac{m_1 m_2}{m_1 + m_2} \approx m_1. \quad (6)$$

In the above equations  $\theta$  denotes the in-plane angle,  $\varphi$  the out-of-plane angle, and  $l$  the distance between the endmasses (tether length) as defined in Fig. 2. The angular rate of the space station in its circular orbit  $\Omega$  is calculated by

$$\Omega = \sqrt{\frac{\mu}{r_0^3}}, \quad (7)$$

where  $\mu = 3.986005 \cdot 10^{14} m^3/s^2$  denotes the gravitational parameter of the earth. The equations of motion of the system are given by

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \left( \frac{\partial L}{\partial q_i} \right) = Q_{q_i}, \quad (8)$$

with  $q_i = \theta, \varphi, l$  for  $i = 1, 2, 3$ , where  $Q_{q_i}$  denotes the generalized disturbing or control force in the  $q_i$  degree of freedom not including gravitational effects. In the present study the out-of-plane motion is not considered, since it is not coupled to the in-plane motion. Thus for a pure in-plane-motion no out-of-plane displacement is to be expected, if no disturbing forces along an inclined orbit are considered. However, the opposite is not true [1]. Therefore, the reduced set of differential equations for the planar motion is given by [6]:

$$\ddot{\theta} = -3\Omega^2 \sin \theta \cos \theta - 2\frac{\dot{l}}{l}(\dot{\theta} - \Omega) \quad (9)$$

$$\ddot{l} = -\frac{F_B}{m_1} + l(\dot{\theta}^2 - 2\dot{\theta}\Omega + 3\Omega^2 \cos^2 \theta) \quad (10)$$

Neglecting the disturbing force components, the only acting force is  $F_B$ , the breaking force provided by the deployment mechanism.

## Control and Identification of Dynamical Systems Using Neural Networks

The application of neural networks to the deployment control of orbital tether systems is of great interest, since the dynamic system is highly non-linear and time-variant. Therefore, a sequential controller has to determine suitable control actions in order to minimize deviations of the tether motion from an optimal reference track. The only control of the system is represented by the breaking force of the deployment mechanism. Two different control strategies using neural networks may be distinguished, the direct and the indirect control strategy. In the present approach the indirect control strategy was investigated.

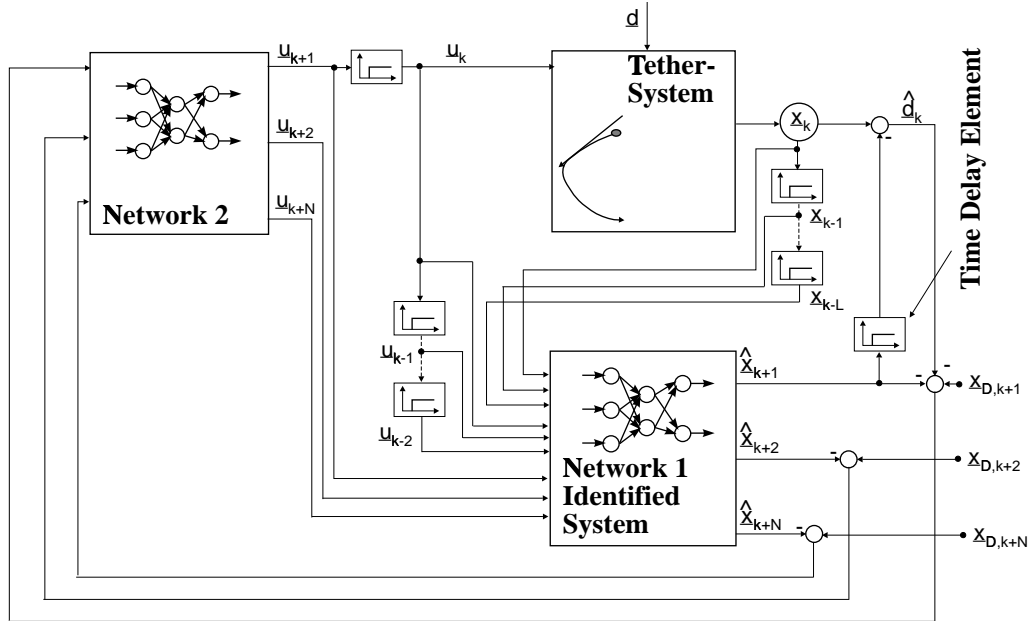


Figure 4: An indirect neural controller [7]

An indirect controller for a dynamic system consists of the following elements [8]:

- A neural network which is trained to represent the dynamic model of the system itself. The network estimates the state-vector for a future time period by using the present state-vector derived from sensor information  $\bar{x}_k$ , as well as the state-vectors from past periods  $\bar{x}_{k-L}$ . Since the control  $u_k$  highly effects the future state of the system, it has also to be considered, i.e. its present, past, and estimated future values.
- A further neural network, Net 2 in Fig. 4, represents an inverse dynamic model of the controlled system. This net acts as a controller. The input variables are the deviations of the estimated state vector  $\bar{x}_{D,k+N}$  from the future reference state vector  $\bar{x}_{D,k+N}$  given by the reference profile. The output therefore corresponds to the control  $\bar{u}_{k+N}$  of the future period.
- means for building the deviations of the estimated state-vectors from the reference state-vectors
- time delay elements

The indirect controller is hence based on the identification of the controlled system. The quality of the controller therefore strongly depends upon the quality of the system identification [7], [9], [10].

The present study deals with the identification of the tether dynamics using a neural network. Since this task is carried out at an operating point, the delayed feedback can be renounced. Fig. 5 shows the network training setup. The training patterns are either obtained directly from the real system (if available) or, as in the present investigation, they can be simulated by integrating the equations of motion representing the dynamic system.

In order to obtain a number of training patterns with differing information on the physics of the problem, a tool is required for the examination of training patterns that show different physical variables but represent identical solutions from a similarity point of view. The use of similar training patterns would have no positive effect on the training of the network [11]. Thus a similarity transformation based on the so-called Buckingham-Theorem

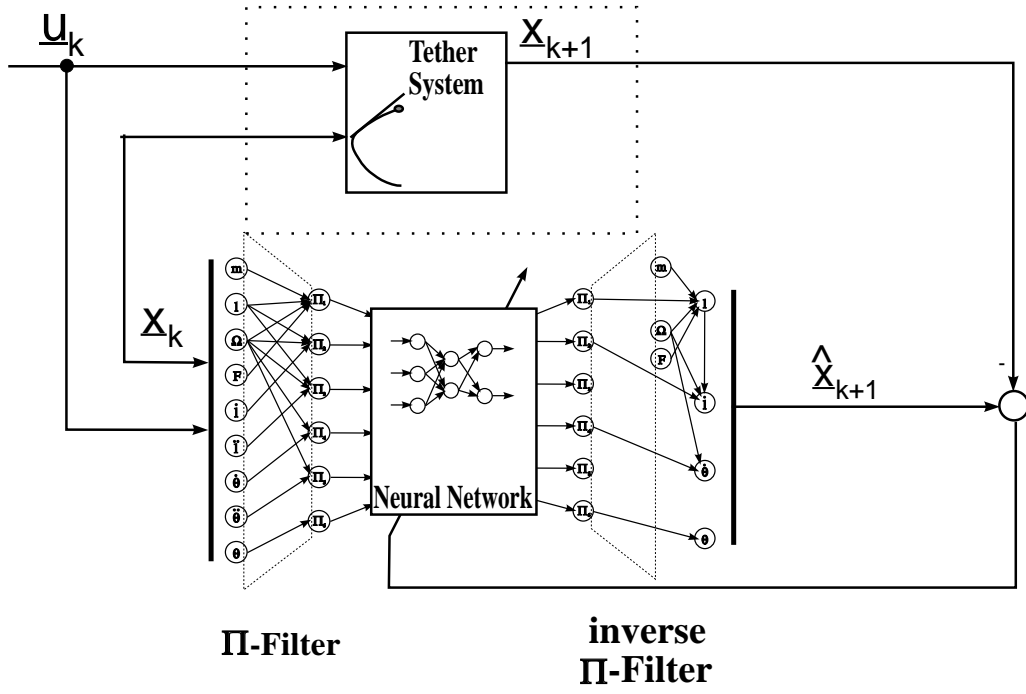
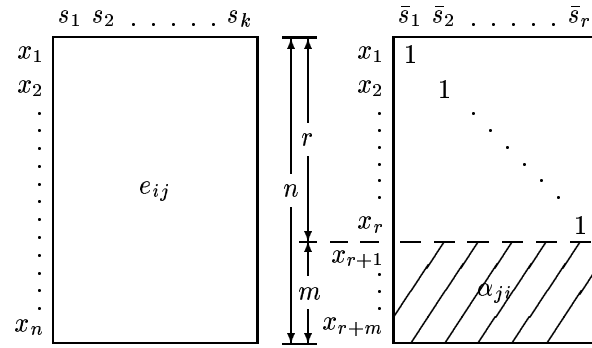


Figure 5: Training setup for system identification

or Pi-Theorem is applied. In this approach dimensionless  $\Pi_j$ -products are built from the physical variables [12]. Similar cases now show identical dimensionless products. By this way, similar cases can be identified and filtered out of the training dataset.

The dimensional matrix is built containing the relevant dimensions  $x_i$  in the mass-length-time fundamental system as shown below according to [11]:



The application of the Pi-Theorem yields

$$\Pi_j = x_j \prod_{i=1}^r x_i^{-\alpha_{ji}}, \quad (11)$$

where  $r$  denotes the rank of the dimensional matrix and  $\alpha_{ji}$  the exponent sub-matrix. The dimensionless products  $\Pi_j$ , with

$$\Pi_1 = \frac{F}{ml\Omega^2}, \quad \Pi_2 = \frac{l}{l\Omega}, \quad \Pi_3 = \frac{\ddot{l}}{l\Omega^2}, \quad \Pi_4 = \frac{\dot{\theta}}{\Omega}, \quad \Pi_5 = \frac{\ddot{\theta}}{\Omega^2}, \quad \Pi_6 = \theta, \quad (12)$$

can be regarded as dimensionless similarity numbers. These dimensionless variables build up the input and the output layer of the neural network in Fig. 5. For details on dimensional analysis see [11] and [12].

# Identification of the dynamic system

## The optimal reference trajectory

The tether is deployed along an optimal trajectory. Due to the advantage of a reduced tether length, a dynamic release of the capsule is considered. The performance index was chosen such, that the control input is minimized. This corresponds to a minimization of the energy added to the system through the control force. The tether is assumed to be initially stabilized in a vertical hanging position, where the initial tether length amounts to  $1000m$  as depicted in Fig. 6. After release, free deployment occurs, thus no tether tension is imposed according to Fig. 7. Due to this fact and since the equations of motion describe the relative motion of the capsule with respect to the space station, the capsule is moving in a slightly elliptic orbit. The tether length increases corresponding to the also increasing deployment velocity as shown in Fig. 6. After passing the perigee at  $t \approx 2750s$ , the deployment velocity decreases again, even before breaking is initiated at  $t \approx 3100s$ . Note that only low tension values are commanded according to Fig. 7. At deployment termination the imposed constraints are met. The final tether length amounts to  $30km$  and the deployment velocity is reduced to  $\dot{l} = 0 m/s$ . Fig. 8 illustrates the deployment path in cartesian coordinates as defined in Fig. 2. This two-point boundary value problem was solved by applying a parameter optimization method [13].

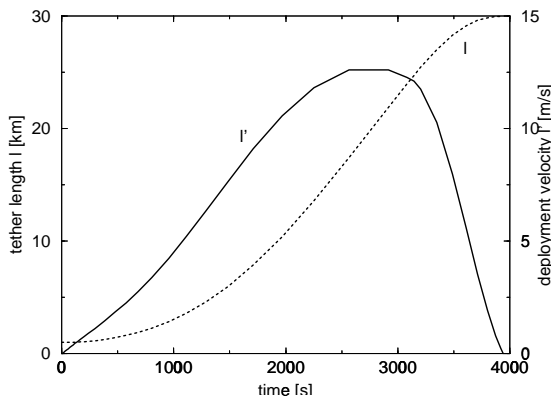


Figure 6: Tether length and deployment velocity vs. time

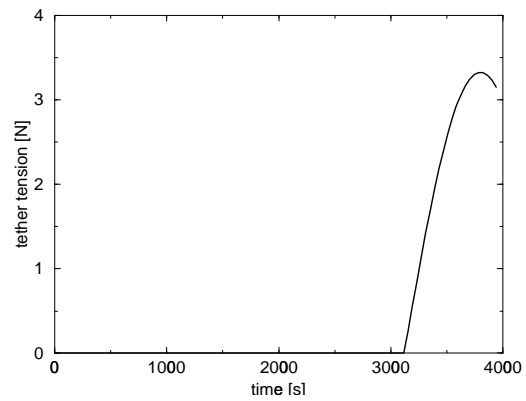


Figure 7: Breaking profile, tether tension vs. time

## The simulation of the training-patterns

In order to obtain the training patterns, the introduced reference profile is used. At an arbitrarily chosen operating point (in this work at  $t = 2000s$ ), disturbances are imposed on the states. They correspond with deviations from the reference accumulated during the deployment towards the operating point. The magnitude of the disturbances is listed in Tab. 1.

Table 1: Disturbances at the operating point

condition	disturbance in percent
in-plane angle $\theta$	$\pm 2$
tether length $l$	$\pm 10$
in-plane angular velocity $\dot{\theta}$	$\pm 4$
deployment velocity $\dot{l}$	$\pm 10 (\pm 2)$

After the disturbances have been added to the state values, each pattern is simulated by forward integration of the equations of motion. The disturbed states at the operating point build the training input, the training output is given by the final state values at deployment termination subject to the optimal breaking profile. The compliance of a fixed time period is important, since the landing accuracy is highly affected by deployment delays [1]. Fig. 8 indicates, that the training space is very large, subject to a very long time period.

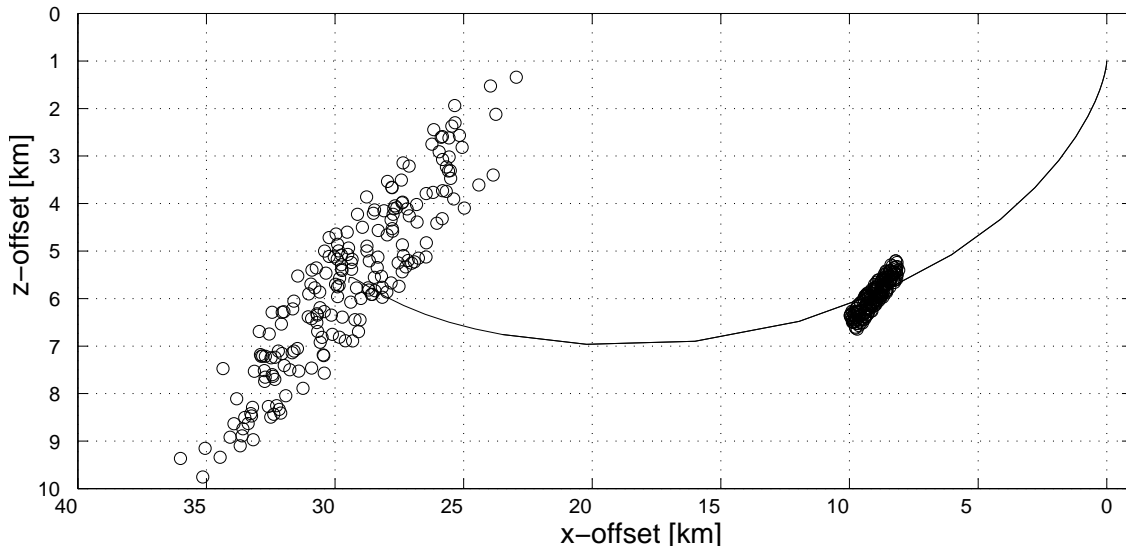


Figure 8: The optimal deployment path with training input and training output data

### The quality of the identification

The trained networks show a simple architecture. In this study, mainly feed-forward networks are applied, providing good results with only one layer of hidden neurons. The activation function chosen is the logistic function. The simplicity of the network topology can be explained by the method of using operating points, by the tether dynamic itself, and by the use of the Pi-transformation. Best training results are obtained by training patterns, that are randomly spreaded over the whole training interval. The results presented in Fig. 9 are subject to a test pattern set containing 50 patterns of a training interval reducing the disturbance of the deployment velocity from 10% to 2% since the training interval is very large. The different columns denote error classes of the end-state vector estimated by the network compared to the exact values obtained by integration of the equations of motion. It has to be considered, that the training space is still very large and that the time period from the operating point up to the deployment termination is long. In the patterns the total tether length varies from  $23\text{ km}$  up to  $34\text{ km}$ , the final deployment velocity from  $-6\text{ m/s}$  up to  $5\text{ m/s}$ . The deviations of the estimated final state vectors from the exact values show a standard deviation of  $363.5\text{ m}$  for the tether length and  $0.145\text{ m/s}$  in the case of the deployment velocity. The identification can thus be considered successful, regarding the size of the training interval.

### Minimizing the network topology

In a further step, the objective is to reduce the arbitrarily chosen network topology to a minimum necessary to provide the same precision of system identification. Therefore, the number of neurons in the hidden layer is reduced step by step, until the post training network error increases. The results show, that a minimum network topology of three neurons in the hidden layer is needed [14]. The same result is obtained by the use of a weight pruning algorithm like the magnitude based pruning [15]. This algorithm first eliminates the weakest links, re-trains and evaluates the pruned network, determines the increase of the error, eliminates the weakest links again, and goes on till the increase of the best error subject to this pruned network reaches a threshold value. The minimum network topology obtained by applying the pruning algorithm is shown in Fig. 10.

### Summary

In the present paper the ability of neural networks to describe the dynamics of orbital tether systems was demonstrated. This was shown for a deployment procedure using a dynamic release. The deployment followed an optimal reference profile, where at an arbitrarily chosen operating point small disturbances were imposed. The training of the neural network took place in a dimensionless space using the Pi-Theorem. A feed forward network containing one hidden layer of 6 neurons was able to estimate the final states at deployment termination within adequate limits. The network topology could be reduced to a minimum network topology using pruning techniques. The results provide the basis for a future development of an indirect adaptive neural controller.

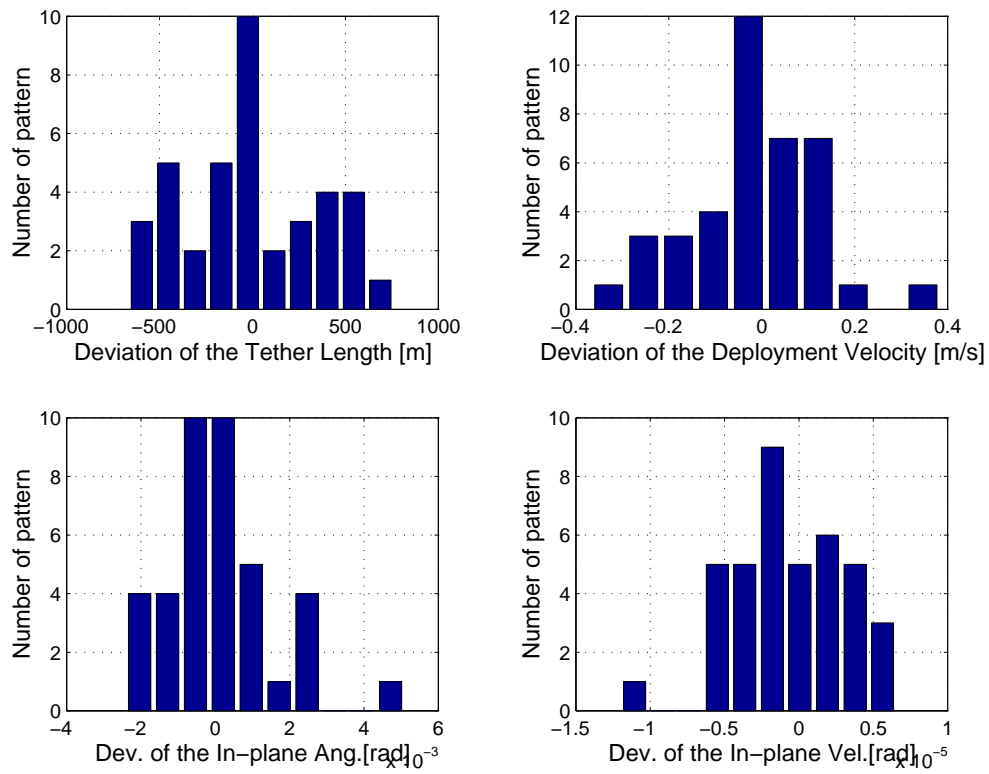


Figure 9: Deviations of the network answer from the exact state values obtained by integration of the equations of motion

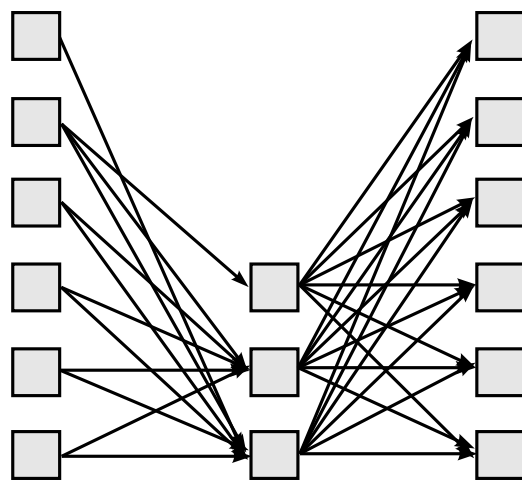


Figure 10: The minimum network topology

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